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HF&VHF Circuits and Techniques I

Solution of Serie 11

Noise Figure, Power Gain and Noise Temperature of an Amplifier

We will assume the following conditions :

- The temperature T is equal to 290 Kelvin.
- The power gain of each amplifier is constant on the frequency bandwidth B = 100 kHz and equals to zero outside this bandwidth B.
- The noise factor of each amplifier is constant on the frequency bandwidth B = 100 kHz.

Question A

- We consider the amplifier 1 such that its power gain in the frequency bandwidth B = 100 kHz is equal to $G_1 = 30$ dB and its noise figure is equal to $NF_1 = 6.02$ dB.
- We consider the amplifier 2 such that its power gain in the frequency bandwidth B = 100 kHz is equal to $G_2 = 10$ dB and its noise figure is equal to $NF_2 = 9.03$ dB.
- Calculate the noise factor F of the circuit constituted by the cascade of the amplifier 1 and of the amplifier 2.

Answer:

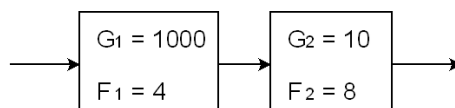
By definition of the gain expressed in dB, we have :

$$G_1 = 10^{\frac{30dB}{10}} = 1000 \text{ and } G_2 = 10^{\frac{10dB}{10}} = 10$$

By definition of the noise figure, we can express the noise factor of the amplifiers as :

$$F_1 = 10^{\frac{NF_1}{10}} = 10^{\frac{6.02dB}{10}} = 4 \text{ and } F_2 = 10^{\frac{NF_2}{10}} = 10^{\frac{9.03dB}{10}} = 8$$

We have the following circuit :



Its noise figure is given by the Friis formula as follows :

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$$F = F_1 + \frac{F_2 - 1}{G_1} = 4 + \frac{8 - 1}{1000} = 4 + \frac{7}{1000} \approx 4$$

Thus the noise figure of the whole cascade is given by :

$$\overline{NF} = 10\log(F) = 10\log(4) = 6.02[\text{dB}].$$

Question B

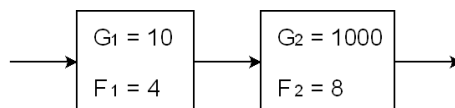
- We consider the amplifier 3 such that its power gain in the frequency bandwidth $B = 100$ kHz is equal to $G_3 = 10$ dB and its noise figure is equal to $NF_3 = 6.02$ dB.
- We consider the amplifier 4 such that its power gain in the frequency bandwidth $B = 100$ kHz is equal to $G_4 = 30$ dB and its noise figure is equal to $NF_4 = 9.03$ dB.
- Calculate the noise factor F of the circuit constituted by the cascade of the amplifier 3 and of the amplifier 4.

Answer:

We have now $G_1 = 10$ and $G_2 = 1000$.

The noise factors of the amplifiers are the same as before: $F_1 = 4$ and $F_2 = 8$.

The circuit is as follows :



As before, its noise figure is given by the Friis formula :

$$F = F_1 + \frac{F_2 - 1}{G_1} = 4 + \frac{8 - 1}{10} = 4 + \frac{7}{10} = 4.7$$

Thus the noise figure of the whole cascade is given by :

$$\overline{NF} = 10\log(F) = 10\log(4.7) = 6.72[\text{dB}].$$

Question C

- Calculate the power gain G of the circuit constituted by the cascade of the amplifier 1 and of the amplifier 2.

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- Calculate the power gain G of the circuit constituted by the cascade of the amplifier 3 and of the amplifier 4.

Answer:

The gain of both cascade is simply given by : $G = G_1 \cdot G_2 = 10^4$ which corresponds to 40 dB. This shows that even if the whole gain is the same, the noise figure change noticeably. As a general rule, it is recommended to have the highest gain amplifiers at the beginning of the cascade (this is only true when noise factors are of the same order of magnitude).

Question D

- Calculate the noise temperature T_r associated to the circuit constituted by the cascade of the amplifier 1 and of the amplifier 2.
- Calculate the noise temperature T_r associated to the circuit constituted by the cascade of the amplifier 3 and of the amplifier 4.

Answer:

Since the noise factor is linked to the noise temperature by the relation (5.17) :

$$F = 1 + \frac{T_r}{T}$$

the noise temperature for case 1) is given by :

$$T_{r1} = (F_1 - 1) \cdot T = (4 - 1) \cdot 290K = 870 K$$

and the noise temperature for case 2) is given by :

$$T_{r2} = (F_2 - 1) \cdot T = (4.7 - 1) \cdot 290K = 1073 K.$$

Question E

- Calculate the Power Spectral Density of the basic noise generated by the circuit constituted by the cascade of the amplifier 1 and of the amplifier 2 by considering this noise at the input of the circuit (notation N_p on p. 5-3 or N_a on p. 5-6 of the HF&VHF course).
- Calculate the Power Spectral Density of the basic noise generated by the circuit constituted by the cascade of the amplifier 3 and of the amplifier 4 by considering this

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noise at the input of the circuit (notation N_p on p. 5-3 or N_a on p. 5-6 of the HF&VHF course).

Answer:

We suppose that the amplifiers are noiseless and that all the noise at output comes from a noise source at the input of the amplifier. The noise factor is linked to the PSD of the noise available at the source by relation (5.16):

$$F = 1 + \frac{N_a}{kT}$$

In case 1), we have :

$$N_a = kT \cdot (F_1 - 1) = (1.38 \cdot 10^{-23} \text{ J/K}) \cdot (290 \text{ K}) \cdot (4 - 1) = 12.006 \cdot 10^{-21} \text{ J}$$

In case 2), we have :

$$N_a = kT \cdot (F_2 - 1) = (1.38 \cdot 10^{-23} \text{ J/K}) \cdot (290 \text{ K}) \cdot (4.7 - 1) = 14.8074 \cdot 10^{-21} \text{ J}$$

Question F

- Calculate the average value of the noise factor F of the circuit constituted by the cascade of the amplifier 1 and of the amplifier 2 on the bandwidth $B = 100 \text{ kHz}$.
- Calculate the average value of the noise factor F of the circuit constituted by the cascade of the amplifier 3 and of the amplifier 4 on the bandwidth $B = 100 \text{ kHz}$.

Answer:

In both cases, the noise factor is constant in the considered bandwidth B . So we simply have :

$$\bar{F} = F$$

Question G

- We assume that the Signal over Noise Ratio at the input (SNR_i) is equal to 10.
- Calculate the Signal over Noise Ratio at the output (SNR_{out}) of the circuit constituted by the cascade of the amplifier 1 and of the amplifier 2.

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- Calculate the Signal over Noise Ratio at the output (SNR_{out}) of the circuit constituted by the cascade of the amplifier 3 and of the amplifier 4.

Answer:

Since the gain of the amplifiers is assumed to be constant on the bandwidth, the signal to noise ratios at input and output are related as follows :

$$\bar{F} = \frac{SNR_i}{SNR_o} \text{ (relation 5.11)}$$

In case 1) we have : $SNR_o = \frac{SNR_i}{\bar{F}} = \frac{10}{4} = 2.5$

In case 2) we have : $SNR_o = \frac{SNR_i}{\bar{F}} = \frac{10}{4.7} = 2.12$

We can notice the degradation of the signal to noise ratio.

Question H

- We assume that the Signal over Noise Ratio at the output (SNR_{out}) is equal to 1. It is recalled that the bandwidth $B = 100$ kHz and that the temperature $T = 290$ Kelvin.
- Calculate the minimum detectable signal (MDS) associated to the circuit constituted by the cascade of the amplifier 1 and of the amplifier 2.
- Calculate the RMS value V_{inRMS} of the input voltage; we do the assumption that the input impedance is equal to $R_{in} = 50 \Omega$.
- Calculate the minimum detectable signal (MDS) associated to the circuit constituted by the cascade of the amplifier 3 and of the amplifier 4.
- Calculate the RMS value V_{inRMS} of the input voltage; we do the assumption that the input impedance is equal to $R_{in} = 50 \Omega$.

Answer:

We have to calculate L_{min} by assuming SNR_{out} is 1, B is 100 kHz and T is 290 K.

Relation 5.26 is expressed by :

$$MDS = L_{min} = \bar{NF} + 10 \log \frac{kTB}{1mW} + (SNR_{out})_{dB} \text{ [dBm]}$$

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Therefore, in Case 1, we have:

$$10 \log \frac{kTB}{1mW} = -124 \text{ [dBm]}$$

$$10 \log (\text{SNR}_{\text{out}}) = 10 \log (1) = 0 \text{ [dB]}$$

$$\overline{NF} = 6.02 \text{ [dB]}$$

$$L_{\text{min}} = -117.98 \text{ [dBm]}$$

The corresponding RMS voltage is calculated as follows :

$$\text{The power is given by : } P_{\text{IN}} = \frac{(V_{\text{IN_RMS}})^2}{R_{\text{IN}}}$$

We also have :

$$L_{\text{min}} = 10 \log \frac{P_{\text{IN}}}{1mW} \Leftrightarrow P_{\text{IN}} = 10^{\frac{L_{\text{min}}}{10}} \cdot 1mW = 1.5922087 \cdot 10^{-15} W$$

which correspond to a voltage of :

$$V_{\text{IN_RMS}} = \sqrt{R_{\text{IN}} \cdot P_{\text{IN}}} = \sqrt{1.5922087 \cdot 10^{-15} \cdot 50} = 0.282153 \mu V$$

In Case 2) we have as above :

$$MDS = L_{\text{min}} = \overline{NF} + 10 \log \frac{kTB}{1mW} + (\text{SNR}_{\text{out}})_{\text{dB}} \text{ [dBm]}$$

$$10 \log \frac{kTB}{1mW} = -124 \text{ [dBm]}$$

$$10 \log (\text{SNR}_{\text{out}}) = 10 \log (1) = 0 \text{ [dB]}$$

$$\overline{NF} = 6.72 \text{ [dB]}$$

$$L_{\text{min}} = -117.28 \text{ [dBm]}$$

We also have :

$$P_{\text{IN}} = 10^{\frac{L_{\text{min}}}{10}} \cdot 1mW = 1.871 \cdot 10^{-15} W$$

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which correspond to a voltage of :

$$V_{\text{IN}_{\text{RMS}}} = \sqrt{R_{\text{IN}} \cdot P_{\text{IN}}} = \sqrt{1.871 \cdot 10^{-15} \cdot 50} = 0.3058 \mu\text{V}$$

Question I

- The circuit constituted by the cascade of the amplifier 1 and of the amplifier 2 is considered. Calculate the output signal level L_{out} when the minimum detectable signal MDS is applied to the input of the circuit.
- The circuit constituted by the cascade of the amplifier 3 and of the amplifier 4 is considered. Calculate the output signal level L_{out} when the minimum detectable signal MDS is applied to the input of the circuit.

Answer:

We simply have: $L_{\text{out}} = L_{\text{min}} + G_1[\text{dB}] + G_s[\text{dB}]$.

In case 1) it gives : $L_{\text{out}} = -117.98 [\text{dBm}] + 30 [\text{dB}] + 10 [\text{dB}] = -77.98 [\text{dBm}]$

In case 2) it gives : $L_{\text{out}} = -117.28 [\text{dBm}] + 10[\text{dB}] + 30 [\text{dB}] = -77.28 [\text{dBm}]$